Further results on the over-all density ratios of shock waves in carbon dioxide

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Previous results on the over-all density ratio of shock waves in CO_2, confirming experimentally the theoretical equilibrium value, have been extended to a shock Mach number of 7·3. The discrepancy between our results and earlier Princeton results approaches 18 % at a Mach number of 7. Possible reasons for this are discussed, with particular reference to the interferometer technique, but no explanation has been found.

Interferometric measurements of the over-all density ratios of shock waves travelling into CO_2 at room temperature were reported in a previous paper (Johannesen, Zienkiewicz, Blythe & Gerrard 1962). The shock Mach number range covered was from 1·4 to 4·1. Using the same experimental technique but changing the shock-tube driver from nitrogen to mixtures of nitrogen and helium, we have now extended the Mach number range to 7·3, and a more detailed comparison with the previous Princeton results is possible. These have recently been discussed in a survey article by Griffith (1961) and we shall use this as reference.

Figure 1 shows the theoretical density ratio corresponding to complete equilibrium together with the Princeton and Manchester measurements. Griffith suggests that the Princeton results indicate that up to a Mach number of 7 the measured density ratio corresponds to the bending mode (of characteristic temperature 959 °K) being in equilibrium and the two stretching modes being unexcited. Above a Mach number of 7·5 the density ratio corresponds to the bending mode and the symmetrical stretching mode (of characteristic temperature 1920 °K) being in equilibrium, and the asymmetrical stretching mode (of characteristic temperature 3380 °K) being unexcited. The Manchester results, on the other hand, correspond over the whole of their range to all modes having reached equilibrium, and the discrepancy between the two sets of results approaches 18 % at a Mach number of 7. This is far outside the expected experimental inaccuracy and it seems likely that one of the groups must have made some fundamental mistake in the evaluation.

A careful check through the whole of our experimental procedure eliminated most of the measurements involved as possible sources of error, but left the effects of impurities and the interferometer technique as requiring further consideration.

As already discussed in our previous paper, impurities were carefully eliminated in the Manchester experiments. In the Princeton experiments they may have been present both in the form of water vapour and as atmospheric air. Water vapour tends to speed up the approach to equilibrium and need therefore not be considered further. Atmospheric air will affect both the thermodynamic properties and the refractive index. As a rough check calculations were made on shock waves at a Mach number of 5 in mixtures of CO_2 and N_2 and it was found that if the Princeton gas had contained 15 % of nitrogen the measured density ratio would correspond to equilibrium. It seems most unlikely that such amounts of air could have been present.

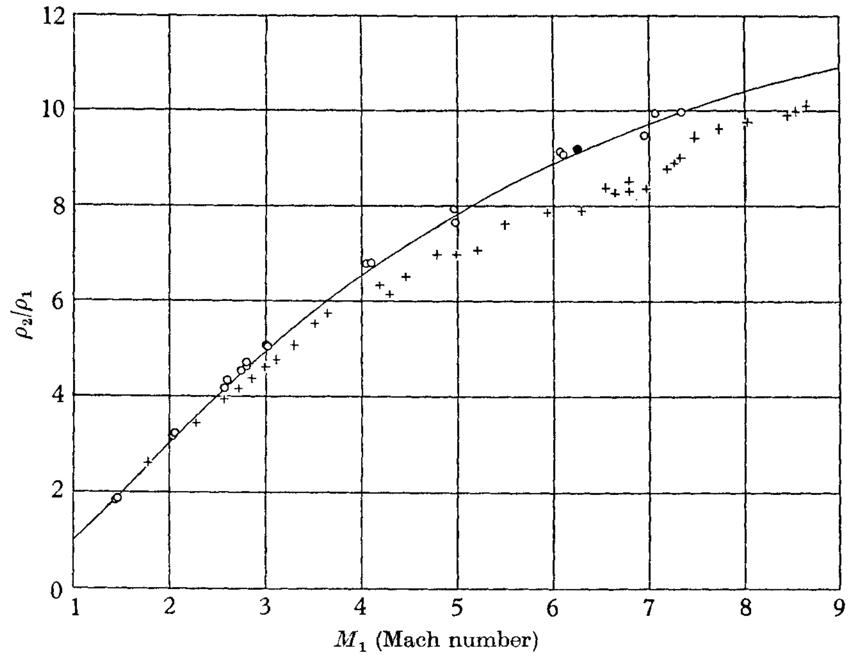

FIGURE 1. Over-all density ratios for shock waves in CO_2. ——, Equilibrium theory; +, Princeton; ○, Manchester (white light); ●, Manchester (monochromatic light).

We are therefore left with the possible sources of error in the interferometer technique, and the main issue here is the effects of dispersion on the appearance of white light fringes.

Consider a situation in which conditions in both beams of the interferometer are identical and the spectrum of the light source consists of a narrow band of width $\delta\lambda$ centred on a wavelength λ. The interference pattern obtained by tilting one of the beams would consist of parallel fringes with spacing proportional to λ. The contrast of the fringes would decrease slowly from the maximum associated with the 'central' fringe, that is the fringe corresponding to zero difference between the optical paths in the two beams. Suppose now that the density of the gas in one of the beams is changed, thereby changing the phase refractive index $\mu = 1 + K(\lambda)\rho/\rho_0$, where K is the Gladstone–Dale factor, ρ is the density of the gas, constant along the light path, and ρ_0 is a reference density. This would result in a bodily shift of all the fringes, each fringe being displaced by an amount proportional to the change of μ. The change of density can be evaluated by measuring the shift of a particular fringe relative to the pattern of fringes obtained with the same setting of the interferometer but using monochromatic light of wavelength λ_m, say. Call D the length, along the light path, of the region in which the density of the gas has been changed from ρ_1 to ρ and suppose that the position of a particular fringe had shifted from that corresponding to the undisturbed monochromatic fringe number N_1 to N. Then†

$$(\rho-\rho_1)/\rho_0 = \lambda_m(N-N_1)/DK(\lambda). \quad (1)$$

The point of using non-monochromatic light is to make one fringe, the fringe of maximum contrast, distinguishable from others. However, whilst individual fringes are displaced according to changes of the phase index μ, the shift of position of the highest contrast is related to the change of the *group* refractive index

$$\mu_g = 1+K_g(\lambda)\rho/\rho_0,$$

where $$K_g = K-\lambda(dK/d\lambda).$$

As the density of the gas is altered by the amount corresponding to a shift ΔN in the position of a particular fringe, the position of maximum contrast will have drifted from one fringe to the adjacent fringe when

$$\Delta N = K/(K_g-K).$$

For CO_2, $K/(K_g-K)$ varies between about 15 in the violet to 35 in the red end of the spectrum. Thus in following the displacement of the fringe of highest contrast as the density of CO_2 is varied, an error of one fringe will be made in the determination of the fringe shift, unless allowance is made for the drift of the position of maximum contrast, when the fringe shift exceeds 7 to 17 fringes, depending on the wavelength of the light used.

These considerations are based on the usual arguments concerning the propagation of a group of waves whose speed depends on the wavelength, and they therefore, strictly, only hold in the limit as the source band width $\delta\lambda \to 0$. However, when dispersion is low, i.e. when $(K_g-K)/K \ll 1$, which is the case for most permanent gases, the concept of group velocity still supplies an adequate description of the formation of fringes even with white light, provided that λ is identified with an effective mean value of the source, λ_e. Therefore, if the drift of the fringe contrast is allowed for, the white-light technique measures changes of the *phase* refractive index at the effective mean wavelength. Then (1) may still be used for evaluating white-light interferograms, provided K is set equal to $K(\lambda_e)$. The actual value of λ_e depends not only on the spectrum of the source but also on the transmission characteristics of the optical apparatus and the colour sensitivity of the eye or photographic emulsion. For sunlight or light from tungsten filaments λ_e is in the green-orange part of the spectrum and varies between 5600 and 5900 Å (see Craven 1945). Spark sources such as the one used in our experiments are much richer in the blue and violet light and one would expect their λ_e to lie in the blue-green. In this part of the spectrum the value of $K/(K_g-K)$ for CO_2 is approximately 20.

In our experiments fringe shifts across shock waves in CO_2 varied from about 18 at $M = 1{\cdot}4$ to 4 at $M = 4{\cdot}1$ and from $4{\cdot}7$ at $M = 5$ to $2{\cdot}3$ at $M = 7{\cdot}3$. In

† In the paper by Johannesen *et al.* (1962) values quoted for λ/DK are the values of DK/λ.

evaluating the interferograms we have recognized and allowed for the drift of fringe contrast, which was significant only at the lower Mach numbers, but we have incorrectly identified $K(\lambda)$ in (1) with $K(\lambda_m)$ rather than $K(\lambda_e)$. Fortunately, K for CO_2 varies by less than 3 % over the visible spectrum. In our case λ_e (as deduced from the spacing of the three white-light fringes of highest contrast) appears to be between 4800 and 5000 Å whilst $\lambda_m = 4425$. As $K(4900) = 4{\cdot}53 \times 10^{-6}$ (when ρ/ρ_0 is in Amagat) and $K(4425) = 4{\cdot}56 \times 10^{-6}$, the error introduced by our wrong choice of K is not likely to exceed $\frac{2}{3}$ %.

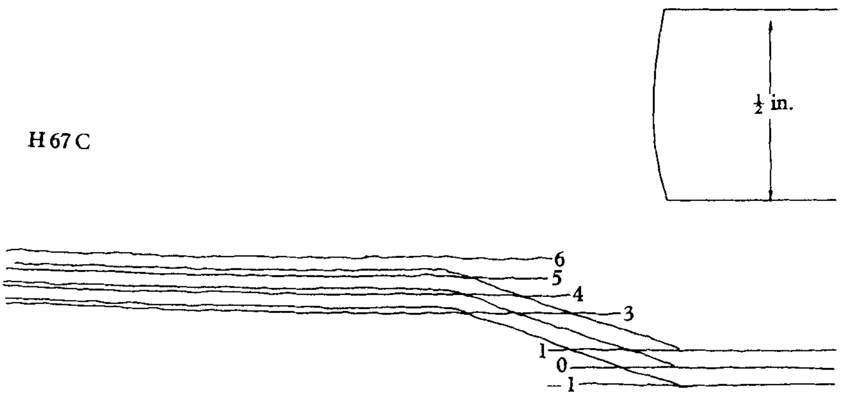

Figure 2. Interferometer record.

As a check on our results based on the white-light fringe technique, a number of monochromatic interferograms were taken of shock waves in CO_2 using an offset beam making an angle of 4 degrees with the plane of the shock. A typical example is shown in figure 2 in which the positions of three disturbed monochromatic fringes ($\lambda = 4435$ Å)† are superimposed on the corresponding undisturbed fringes. The shock was travelling at a Mach number of 6·26 into dry CO_2 at 23 °C and 2·03 mm Hg. Under these conditions the final equilibrium density ratio is theoretically 9·11 as compared with a measured value of 9·17. The measured fringe shift is 4·20. Had only the bending mode been excited this would have been about 3·4. This check thus confirms the results obtained with the white-light technique. It is worth noting that figure 2 includes a very considerable length of uniform density behind the shock wave.

In conclusion we can only repeat that the large discrepancy between the Princeton and Manchester results remains unresolved.

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† A slightly different filter was used in these experiments.